Math 220, section 31
Spring 2001
Exam No. 2

Problem 1 Perform computations or provide short proofs as required. (50 points)

(i) The potential energy of a double pendulum is given by $V(\theta_1, \theta_2) = (m_1+m_2)gl_1(1-\cos\theta_1)+m_2gl_2(1-\cos\theta_2)$. Find the angles $\theta_{1,2}$ corresponding to the minimum, and write all the terms up to order $\theta_{1,2}^2$ in the power series expansion for V about its minimum.

(ii) If x and y are given by

$$x = \int_u^{\cos(v)} e^{-t^2} dt \text{ and } y = \int_0^v (u + x + t)dt,$$

find $(\partial y/\partial x)_u$. Variables u and v are independent.

(iii) Find the area of the surface obtained by rotating the segment of the curve $y = x^2$, $0 \leq x \leq 3$, about the y axis.

(iv) Show that the vector field $\mathbf{F} = (2xy + 3yz)\mathbf{i} + (x^2 + 3xz)\mathbf{j} + 3xy\mathbf{k}$ is conservative.

(v) If D is a domain in the plane, and ∂D is its boundary, show that

$$\oint_{\partial D} xydy = \int\int_D ydxdy.$$

Use this formula, along with symmetry, to calculate the centroid of a lamina that is given by $x \geq 0, y \geq 0, x^2 + y^2 \leq a^2$.

Problem 2 A pair of coupled oscillators are governed by the equations

$$\begin{aligned}\ddot{x}_1 &= -2x_1 + x_2 \\ \ddot{x}_2 &= 2x_1 - 3x_2\end{aligned} \tag{1}$$

Write this system in the matrix form $\ddot{\mathbf{x}} = A\mathbf{x}$. Guessing a solution with an exponential time dependence $\mathbf{x}(t) = \mathbf{v}e^{\omega t}$, find the allowed values of ω and the corresponding vectors $\mathbf{v}$. Write the general solution for this system, taking into account that x_1 and x_2 are real. (30 points)

Problem 3 A dielectric cylinder sits in the region $0 \leq z \leq 2$ and $x^2+y^2 \leq 9$, and there are no charges outside this cylinder. If the free charge *surface* density on the cylinder is given by $\rho(x, y, z) = \sqrt{x^2 + y^2} + 4x^2 e^z$, find the total flux of the electric field through the sphere $x^2 + y^2 + z^2 = 25$. (Hint: Use cylindrical polar coordinates. Also, $\int_0^{2\pi} \cos^2\theta d\theta = \pi$) (20 points)